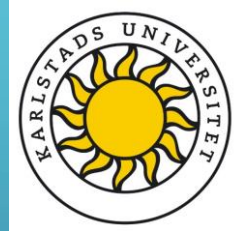


COMBINING A DYNAMIC MATHEMATICS SOFTWARE AND A COMPUTER-AIDED ASSESSMENT SYSTEM TO ENCOURAGE ENGINEERING STUDENTS' MATHEMATICAL THINKING

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***THE 1ST NORTHERN E-ASSESSMENT MEETING
TRONDHEIM, 31 MAY–2 JUNE, 2023***

 **NTNU**

 **MatRIC** Centre for Research,
Innovation and Coordination
of Mathematics Teaching

BACKGROUND

- Mathematics is the first subject the engineering students encounter when they begin their university studies. The transition from high school mathematics to university mathematics represents a major challenge for many students, both in Sweden and internationally.
- The student groups are large and heterogeneous, while many students lack suitable study techniques.
- The university studies demand self-responsibility for the study situation.

TRADITIONAL WAY OF TEACHING MATHEMATICS AT UNIVERSITY:

MOST OF THE FOCUS IS ON LECTURES & SEMINARS
LESS FOCUS IS ON STUDENT'S OWN WORK (ALONE OR IN GROUPS)



AN ALTERNATIVE COURSE DESIGN



Teacher's teaching room



Peer teaching and learning room



Self-studies room

Context:

- * first-year engineering students taking the first math course in a large group
- ** long-time low scores in exams independent of the teacher and high-level of dropouts

2015



Mats



Mirela



Maria

Considerable
increased workload
for the teacher

Mandatory
small group
activities in
GeoGebra

Let's use our experience
from work with
inquire-based
tasks in DMS



Mirela



Mats

New project 2020



Maria



Yosief

BACKGROUND

- Continuous assignments have become increasingly common thanks to an intensive development of digital solutions in the form of computer aided assessment systems, where students receive direct feedback. An identified risk with this type of assignments is, however, that there is too much focus on procedural tasks as they are easier to autocorrect.
- In addition, the feedback given is often limited to whether the answer is right or wrong, sometimes with the correct answer added. In this way, students are not offered feedback that can function **formatively**.
- In order for feedback to be effective, and thereby support students' learning, research shows that **it must also contain some form of additional information** for the student.

GOAL AND METHOD

- The purpose of the project is to develop principles for the design of digital learning environments in the form of exploratory activities with customized automatically generated **formative feedback**.
- To make this possible, we combine a computer-aided assessment system (Möbius) with a dynamic mathematics software (GeoGebra).
- This requires carefully thought out assignments and well-crafted feedback based on student response. Although there is a lot of research on both of these digital tools individually, there are still only few studies focusing on a combination of them.
- By using a design-based approach where the designed digital learning environment is evaluated and developed on repeated occasions, we develop general design principles.
- Empirical data is collected through surveys and analysis of student work where we examine how first-year engineering students use the different types of feedback.

CONTEXT

- Three cohorts (2020, 2021 and 2022): 256 + 235 + 224 first-year engineering students taking the first course in Calculus
- As part of the course assignment: two computer-based compulsory small group activities (designed for combined use of a DMS environment and a CAA system)
- Each activity consists of a sequence of tasks (focusing on functional understanding), where the tasks:
 - - require either a group response or an individual response
 - - are mainly of three different non-traditional forms:
 - Exploratory activities (describe/explain)
 - 'Translation' tasks (from-graph-to-formula tasks)
 - Example generating tasks

ONGOING PROJECT: INQUIRY-BASED GROUP ACTIVITIES



Task structure
Investigate, by using GeoGebra, how the graph of the trigonometric function $f(x) = A \sin(B(x - C)) + D$ depends on the values of the parameters A, B, C and D.

Group answer:
Describe in what way the various parameters alter the graph.

Discuss within the group how $f(x) = A \sin(B(x - C)) + D$ depends on A, B, C and D.

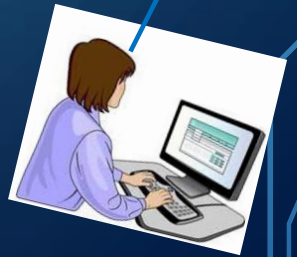
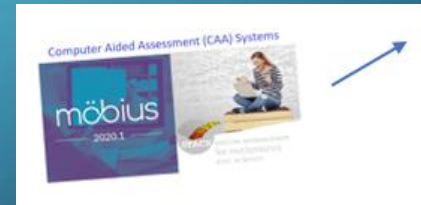
The figure below shows $y = |f(x)|$. In this task, you are expected to use knowledge from the course to solve an applied problem. (see figures below) to be replaced. Your task is to determine two different functions f and g that can be used during construction so that there is a smooth transition in points A and B.

Note! Group members may be asked to use the measurements in the sketch to the right below.

Individual answer:
 $f(x) =$ _____
 $g(x) =$ _____

Group answer:
 1) $f(x) =$ _____
 2) $g(x) =$ _____

Group answer:
Describe what knowledge from the course you used to solve the problem.



**Combining a dynamic mathematics software and
a computer-aided assessment system**
- to encourage engineering students' mathematical thinking

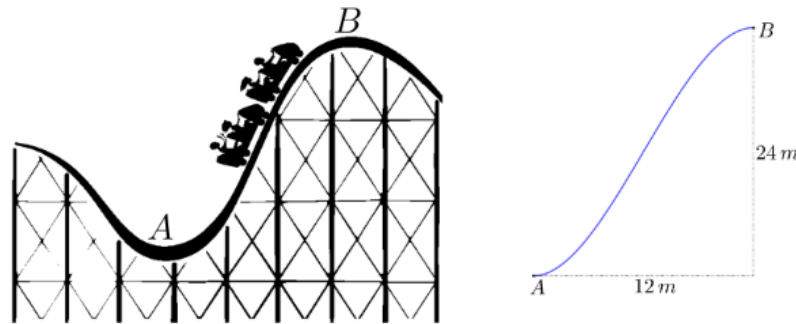
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Karlstad University, Sweden

The 1st Northern e-Assessment Meeting
Trondheim, 31 May – 2 June, 2023

Example 1 (Exploratory activity, mathematical modeling)

Part of a structure in a roller coaster need to be replaced. It is the part between point *A* and point *B* (please see the figures below) that must be replaced. Your task is to determine a **polynomial function** f that can be used during the construction in such a way that there is a smooth transition at point *A* and point *B*.

- Use the measurements in the sketch at the right.
- Use GeoGebra to check whether you have come up with a suitable function before entering it as an answer to the task.



Group agreed response:

$f(x) =$

Use the "Verify" button below to check if your response is correct. If not, you will get some clues. You then get another chance to redo the task.

Note: After you have pressed the "Verify" button, you will not have another chance to change your answer!

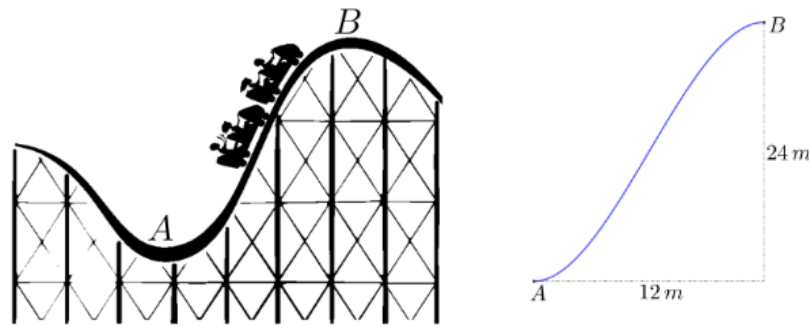
Section Attempt 1 of 1

Verify

Example 1 (Exploratory activity, mathematical modeling)

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- Use the measurements in the sketch at the right.
- Use GeoGebra to check whether you have come up with a suitable function before entering it as an answer to the task.



Your response does not seem to match the correct answer:

- Start by placing the points *A* and *B* in a coordinate system with point *A* at the origin.
- As suggested, form one such polynomial function (with so far unknown coefficients).
- Use the conditions that $f(0) = 0$, $f(12) = 24$, $f'(0) = 0$ and $f'(12) = 0$ to determine the coefficients.

Use these hints and try again!

Group agreed response:

$f(x) =$   

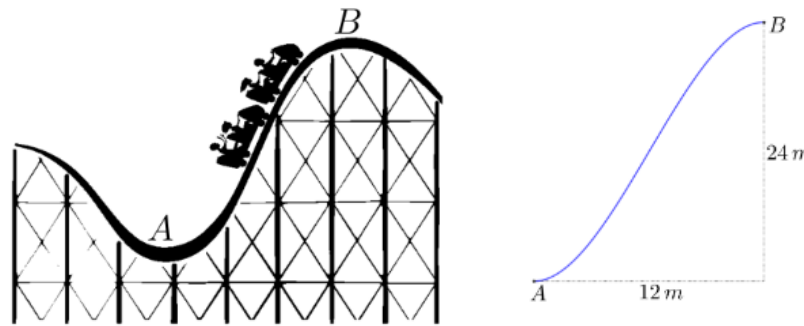
Section Attempt 1 of 1

Verify

Example 1 (Exploratory activity, mathematical modeling)

You will now solve the same task once more, but this time you will determine a **trigonometric function** g that can be used during the construction so that there is a smooth transition at point A and point B .

- Use the measurements in the sketch at the right.
- Use GeoGebra to check whether you have come up with a suitable function before entering it as an answer to the task.



Group agreed response:

$g(x) =$

Use the "Verify" button below to check if your response is correct. If not, you will get some clues. You then get another chance to redo the task.

Note: After you have pressed the "Verify" button, you will not have another chance to change your answer!

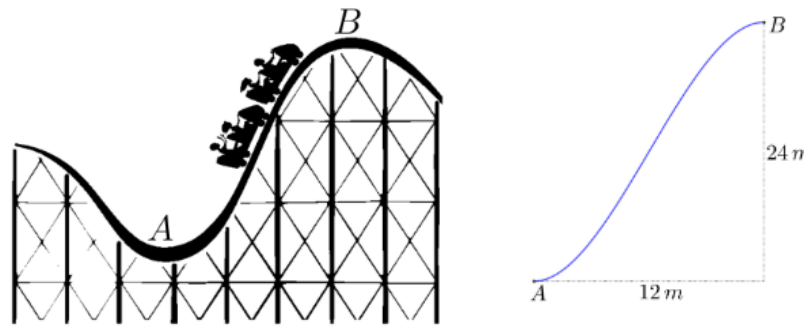
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Example 1 (Exploratory activity, mathematical modeling)

You will now solve the same task once more, but this time you will determine a **trigonometric function** g that can be used during the construction so that there is a smooth transition at point A and point B .

- Use the measurements in the sketch at the right.
- Use GeoGebra to check whether you have come up with a suitable function before entering it as an answer to the task.



Your response does not seem to match the correct answer:

- Start by placing the points A and B in a coordinate system with point A at the origin.
- Determine the amplitude, the period, and the shifts in the x - as well as the y -direction, so that the function fulfills the requirements.

Use these hints and try again!

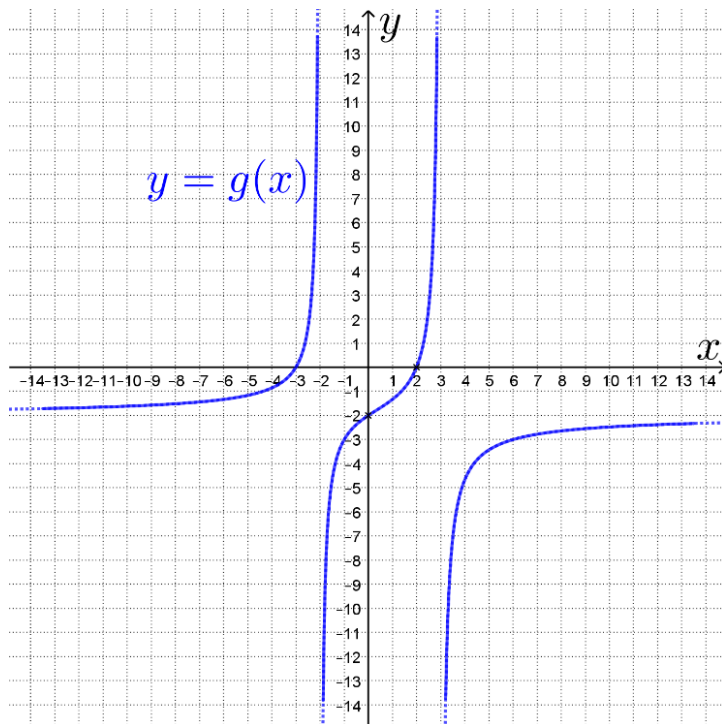
Group agreed response:

$$g(x) = \text{[input box]}$$

Section Attempt 1 of 1

Verify

Example 2 (Representing a function)



The figure above shows $y = g(x)$, graph of the function g . Use the graph to determine the function formula for g .

- Use GeoGebra to check whether your response is correct, before entering it as an answer to the task.

Group agreed response:

$$g(x) = \text{[input field]}$$

Use the "Verify" button below to check if your response is correct. If not, you will get some clues. You then get another chance to redo the task.

Note: After you have pressed the "Verify" button, you will not have another chance to change your answer!

Section Attempt 1 of 1

Verify

Your response does not seem to match the correct answer:

The following is a silent recording on a similar task. It illustrates how the given asymptotes are used to form the function.

Make use of these hints and try again! Use the same graph (the one at the top of the page) to determine the function formula for g .

Group agreed response:

$$g(x) = \text{[input field]}$$

Section Attempt 1 of 1

Verify


Example 3 (Give your own examples)

Give examples of two different functions, f and g , that both have

- two vertical asymptotes at $x = -7$ and $x = 2$, as well as
- one horizontal asymptote: $y = 3$.

- **Note:** Group members may have obtained different asymptotes.
- Use GeoGebra to check whether each of the functions in your response really have the given asymptotes, before entering them as an answer to the task.

Individual response:

$f(x) =$ 

$g(x) =$ 

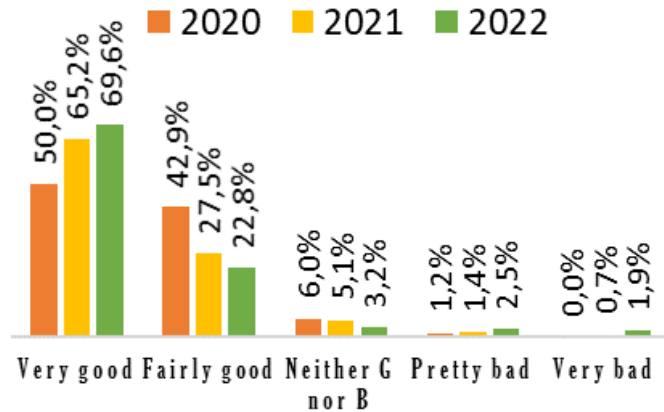
What do the students themselves say?

Surveys (questionnaire) are conducted, with the following response frequency

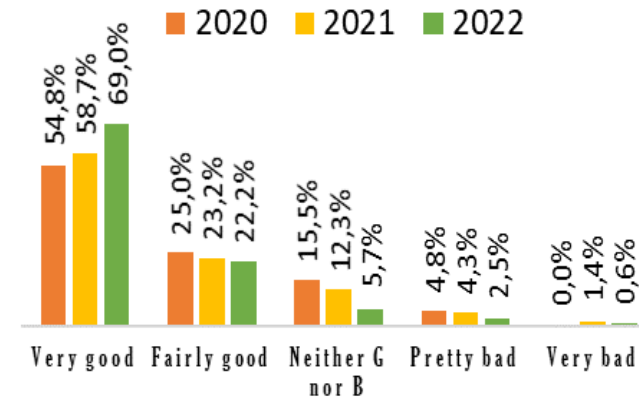
- The autumn term of 2020: 32.8% (84 of 256)
- The autumn term of 2021: 59.1% (139 of 235)
- The autumn term of 2022: 71.9% (161 of 224)

How well do the following statements (*about the group work*) agree with your view?

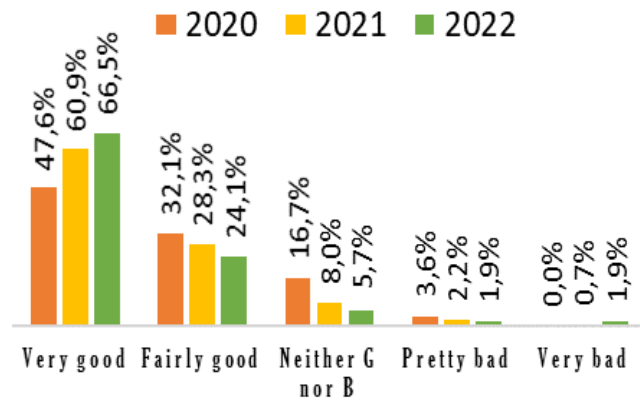
The collaboration within the group worked well.



/Everyone in the group has participated actively in the discussions.

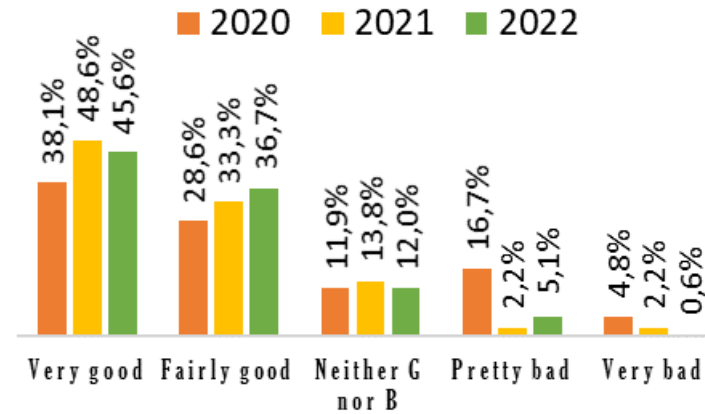


It was instructive to take part in other people's thoughts.

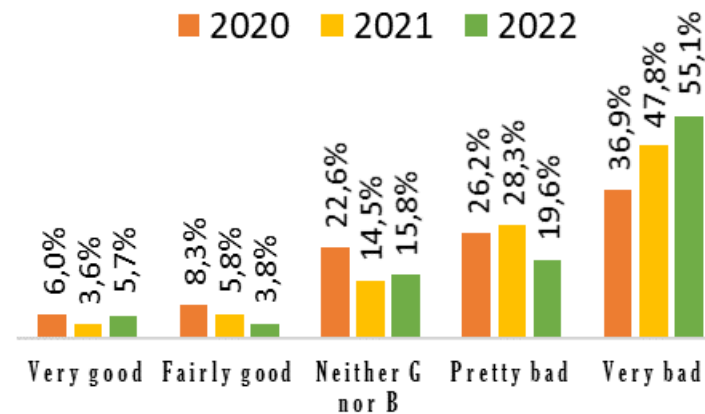


How well do the following statements (*about the group work*) agree with your view?

We also discussed the tasks that required individual response.

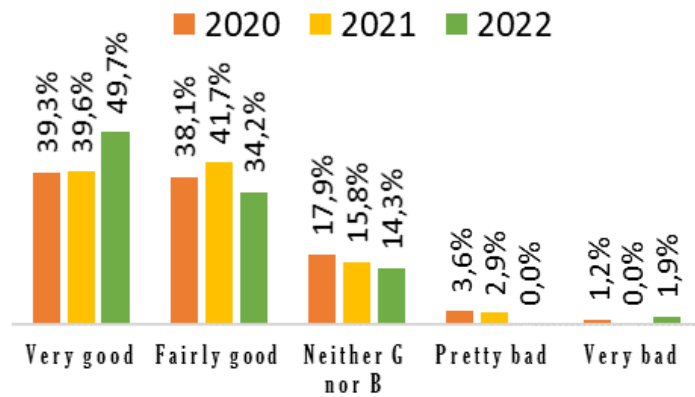


I would have preferred to work on the tasks all by myself.

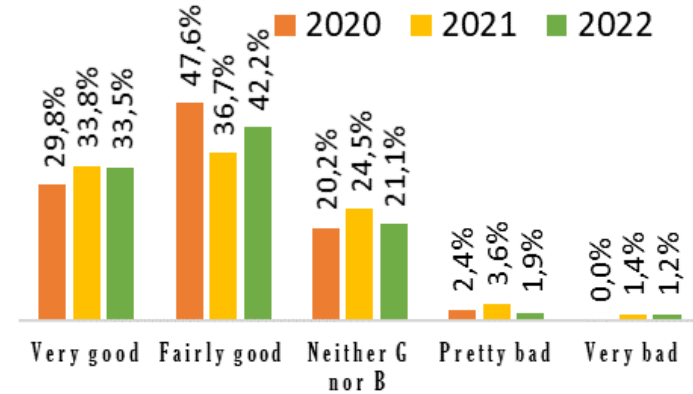


How well do the following statements (*about the tasks*) agree with your view?

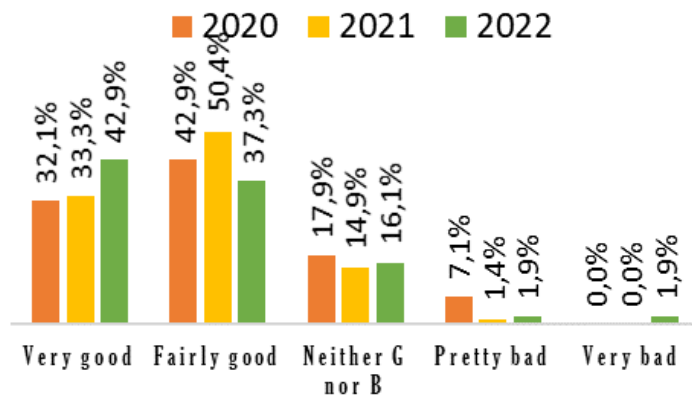
Examining mathematical relationships in GeoGebra and formulating conclusions gave increased understanding.



The work of generating two examples of functions that fulfill given conditions gave increased understanding.

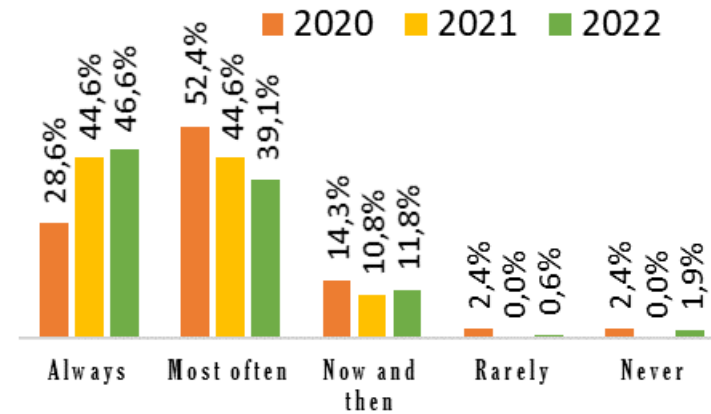


The work of determining the function formula based on a given graph gave increased understanding.



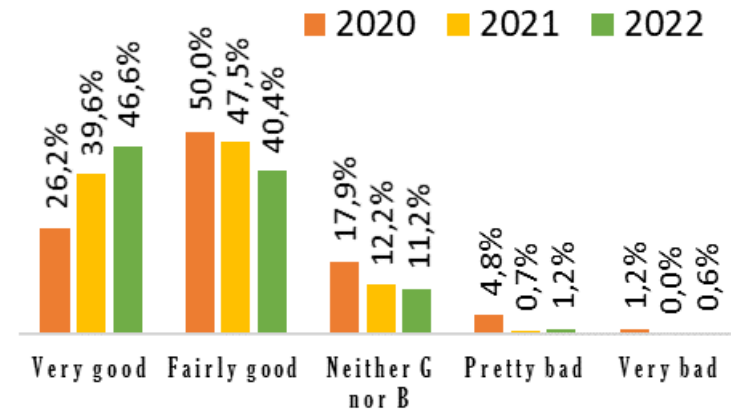
How well does the following statement (*about GeoGebra and Möbius*) agree with your view?

To what extent did you use GeoGebra (when possible) to check your response before entering it into Möbius?

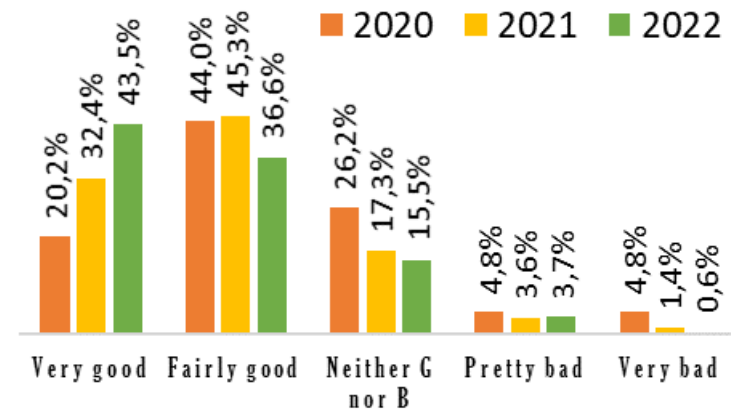


Some summarizing statements: How well do the following statements agree with your view?

The work with the tasks has given me a deeper understanding of the mathematics they deal with.



The time I spent on the tasks was time well spent.



Some concluding remarks

Extended opportunity to provide feedback (in different forms and at different times), e.g.

- the direct/visual response while working with GeoGebra
- possibility to check whether the result is correct (while working on the tasks), or as a solution manual afterwards (delayed feedback)
- the activities are carried out in groups, the group's discussion is a form of feedback
- during the oral examination at the end of the course, where the group's common answers are discussed

There is always some more to keep exploring, specially

- the adaptive functionalities of the CAA system,
all to support engineering students' mathematical thinking

Thank you
for your attention

